On a three dimensional micromechanical damage model and its application to brittle rocks

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ABSTRACT : A general 3-D micromechanical damage model, based on a frictional sliding crack mechanism, is formulated in rate form. First we summarize the main physical and theoretical aspects of the model. Then we present an application on a French sandstone. It is concluded that the model describes the salient features of anisotropic damage of brittle materials : load-induced anisotropy, positive dilatancy, hysteresis loops, non negligible permanent strains, damage deactivation due to microcracks closure.

1 INTRODUCTION

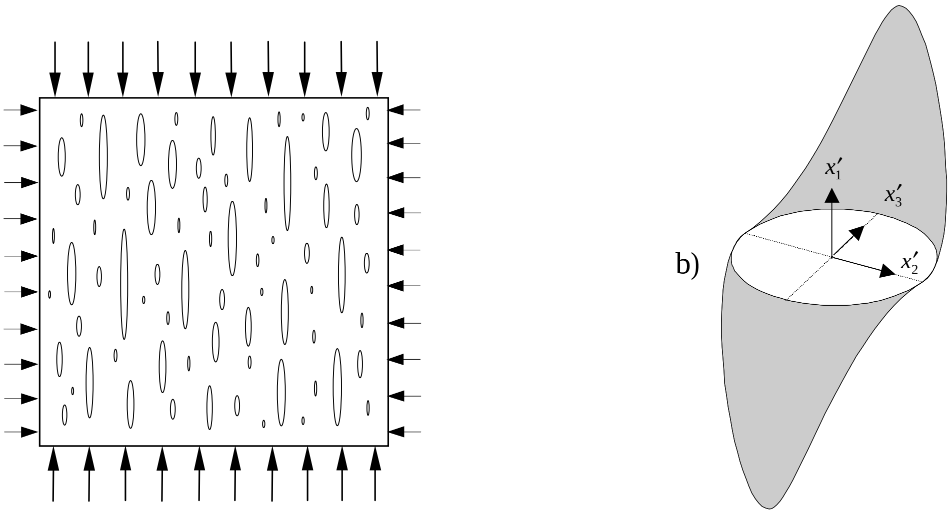
Behaviour of brittle materials (some rocks, concrete, ceramics, etc..) under compressive loading is closely related to growth of microcracks (Kranz, 1983 ; Nemat-Nasser and Horii, 1993). This microcracking phenomenon induces, at the macroscopic scale, complex characteristics among which load-induced anisotropy, dilatancy, friction phenomenon, cracks closure effects (damage deactivation) etc.. Continuum Damage Mechanics (see Krajcinovic, 1996) has been recognized as one of the appropriate tool for the study of such behaviour. The phenomenological approach of damage modeling, based on the use of internal variables, has made since two decades important progress by trying to take into account some of the fundamental aspects of quasi brittle damage (Halm and Dragon, 1998). In spite of these progress the need of more physical approach, based on direct account of deformation mechanisms at the mesoscopic level, remains an important topic in damage modeling. Various works has been devoted to such mesomechanical modeling. But most of them are generally limited to conventional triaxial loading (Krajcinovic and Fanella, 1988 ; Lee and Ju, 1993) or by their basic physical mechanism (Gambarotta et Lagomarsino, 1993) or their two dimensional aspect (Nemat Nasser and Obata, 1988). The main objective of the present study is to develop a 3-D mesomechanical model, formulated in a rate form. The approach is closely similar to the study of Nemat Nasser and Obata (1988) in 2-D, revisited recently by Basista and Gross (1998). Applications performed on a sandstone allow to evaluate the capabilities of the model.

2 PHYSICAL BASIS OF THE 3-D MICROMECHANICAL MODEL

Many observations carried out on brittle materials such as rocks or concrete showed that failure develops by axial splitting due to the growth of axial tension microcracks (Peng and Johnson, 1972). It is generally suggest that the tensile microcracks (see figure 1a) can originate from various mechanisms among which the presence of microcavities, or the pre-existence of inclined microcracks. The first mechanism has been considered in Ashby and Hallam (1986) for a damage modeling.

The second mechanism (frictional sliding crack and branching mechanism, depicted on figure 1b) has been considered in several studies (Fanella and Krajcinovic, 1988 ; Lee and Ju, 1991). It appears that, unlike to models related to the presence of microcavities, the models based on the sliding crack mechanism can be viewed as a viable physical model for the study of inelastic deformation of brittle materials. Moreover, recent results from acoustic emission tests do suggest that both tensile and shear events occur during brittle rock deformation (Lockner, 1993). Therefore, our study considers also the frictional sliding crack mechanism. The approach is similar to the one proposed by Nemat-Nasser and Obata (1988) in 2-D. The present development aims to a generalization of their study to the 3-D case.

a) b)

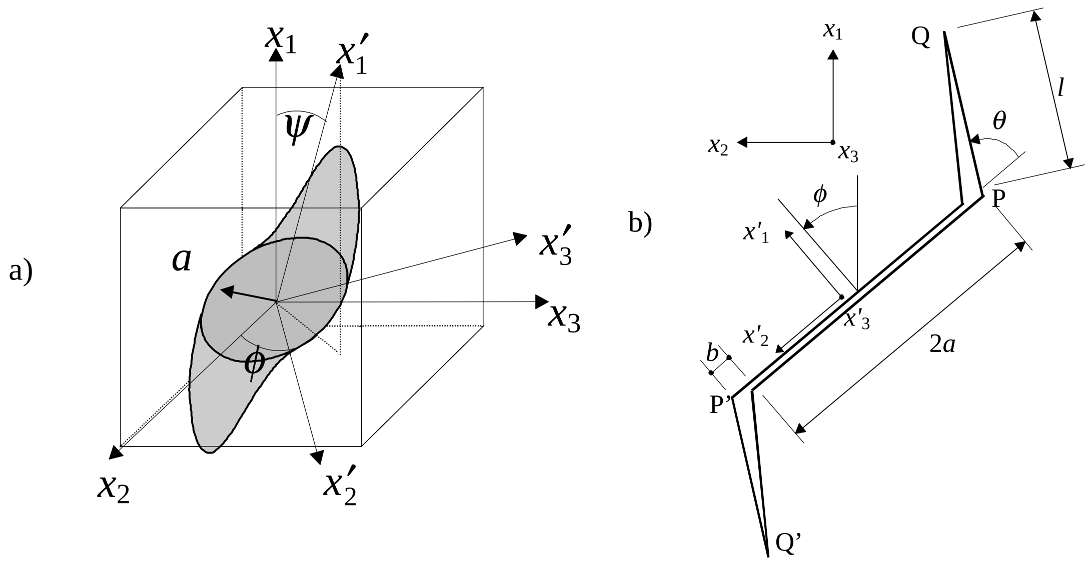


Figures 1 : a) Brittle failure by tensile axial fractures b) 3-D Sliding crack model

3 RATE FORM OF THE 3-D MICROMECHANICAL MODEL

3.1 Strain analysis

The model construction follows the general steps of homogenization of materials with random microstructure : representation, localization, homogenization. For the first step the material is viewed as a two phase medium, constituted by the solid matrix and microcracks. A representative volume element (RVE) containing a great number of penny shaped microcracks is then considered. Moreover, we consider moderate densities of non interacting microcracks. This permits to analyze the deformation of the whole RVE by studying a isolated microcrack (with radius a) in the solid matrix (see figure 2a). Since no closed form solution exists for 3-D branched cracks, we need to simplify the physical model already shown at figure 1b. The basic idea of this simplification follows the proposition of Kachanov (1982), used also by Fanella and Krajcinovic (1988) . The 3-D branched crack is approximated by a serie of 2-D cross sections. Following then Nemat-Nasser and Obata (1988), the sliding crack is described by 3 parameters which are sliding b (two components) on PP', wing crack length l and orientation θ (see figure 2b). In fact, the first two parameters are normalized by the microcrack radius a : $\tilde{b} = b / a$, $\tilde{l} = l / a$.



Figures 2 : a) isolated 3-D penny shape microcrack in the RVE ; b) Geometrical and mechanical representation of the branched microcrack (plane view).

The next step of the model construction is the analysis of deformation of the RVE. Assuming that the matrix is homogeneous, isotropic and behaves linear elastically (with compliance $\bar{\bar{S}}^0$), it can be easily demonstrated (see for example Nemat-Nasser and Horii, 1993) that the overall strain of the RVE is given by :

$$\bar{\varepsilon} = \bar{\bar{S}}^0 : \bar{\sigma} + \bar{\varepsilon}^d \qquad \text{with} \qquad \bar{\varepsilon}^d = \frac{1}{2}\frac{N}{V}\iint_S (n \otimes D + D \otimes n) dS \tag{1}$$

This result is obtained in the case of homogeneous traction $\bar{\sigma}$ applied at the boundary of the RVE. $\bar{\varepsilon}^d$ is the overall inelastic strain due to the presence of all microcracks. N is the number of microcracks in the RVE (volume V). n represents the normal of each microcrack whose area is denoted by S. D is the displacement discontinuity of a current microcrack.

The contribution of a single sliding and branched microcrack to the deformation of the RVE can be decomposed in two parts, one due to the inclined part PP’ of the microcrack, and a second one due to the wing part. It can be then shown that (Renaud, 1998) :

$$\bar{\varepsilon}^d = \omega \left\{ \frac{1-\nu_0{}^2}{3E_0} \tilde{l}\sqrt{\tilde{l}} \left[4\bar{\alpha} \otimes \bar{\alpha} + \bar{\beta} \otimes \bar{\beta} + \bar{\delta} \otimes \bar{\delta} \right] : \bar{\sigma} + \pi \tilde{b} \bar{p}_0 + \tilde{l}\,\tilde{b}\,\bar{q}_0 \right\} \tag{2}$$

where $\omega = Na^3 / V$ is the Budiansky microcrack density. E_0 and ν_0 are the elastic coefficients of the matrix. Second order tensors $\bar{\alpha}$, $\bar{\beta}$ $\bar{p}_0$ and $\bar{q}_0$ depends on orientation in space (angles ϕ and ψ) of the considered microcrack. The term due to $\tilde{b}$ includes the two components of tangential discontinuity.

Finally, homogenization step consists to take the average of the strain tensor over all microcracks. Assuming that microcracks orientations are uniformly distributed the average strain due to microcracks is obtained by integration on all orientations of space :

$$\frac{1}{2\pi}\int_0^{\pi/2}\int_0^{2\pi} \bar{\varepsilon}^d (\psi, \phi) \sin\phi \, d\psi \, d\phi \tag{3}$$

3.3 Rate formulation of the 3-D micromechanical model

The objective here is to provide a general rate formulation of the model. We need then to relate the increments of the kinematics variables, $\dot{b}$, $\dot{l}$ and $\dot{\theta}$ to the increment of applied load. The analysis follows in a straight manner Nemat-Nasser and Obata (1988)'s calculation. Cracks growth is controlled by linear elastic fracture mechanics criteria. Friction is taken of Coulomb type. The calculation follows three steps :
i) stress intensity factors at the front of the wing microcrack is calculated in two ways, one based on forces and the other on displacement discontinuities,
ii) the obtained results are then differentiated,
iii) assuming that the increment of the two stress intensity factors are collinear, a third equation is established.

Resolution of the three equations gives the relations :

$$\dot{\tilde{l}} = \overline{A}:\dot{\overline{\sigma}}\ ,\ \dot{\theta} = \overline{B}:\dot{\overline{\sigma}}\ ,\ \dot{\tilde{b}} = \overline{C}:\dot{\overline{\sigma}} \qquad (4)$$

where $\overline{A}$, $\overline{B}$ et $\overline{C}$ are second order tensors which depends on the stress level and on the load increment. The rate form of the constitutive can then be obtained by differentiating equation (2) :

$$\dot{\overline{\varepsilon}} = \overline{\overline{L}} : \dot{\overline{\sigma}} \qquad (5)$$

4 APPLICATION TO THE ANALYSIS OF THE BEHAVIOUR OF A SANDSTONE

In order to evaluate the suitability of the proposed model, an application is performed on Fontainebleau Sandstone (France). We present here results for a uniaxial cyclic compression and for a progressive cracks closure test. The values of model parameters used in this simulation are

- elastic coefficients E_0 = 39300 MPa, ν_0 = 0.13 ; friction coefficient μ = 0.6 ;
- cohesion $\tau_c^0 = 5\,\text{MPa}$; material toughness : K_I^c = 0.6 MPa$\sqrt{\text{m}}$.
- initial mesocracks density : ω_0 = 0.06. Since for the studied sandstone, any information on the initial microcracks density ω_0 is available, identification of this parameter was not possible. However, the choice ω_0 = 0.06 allows to reproduce data for monotonous compression tests.

4.1 Uniaxial cyclic compression :

Figure 3 shows the model prediction for uniaxial cyclic compression (4 cycles). During loading, the more favorable microcracks branch and propagate in axial direction. Opening of these microcracks generates an anisotropic behaviour which is accompanied by a great dilatancy. The unloading reveals two successive steps : i) linear phase with a compliance greater than the initial one (because of the damaging effect of already propagated microcracks) ; ii) backsliding, in which the microcracks faces sliding is mobilized in opposite sense. Complete unloading indicates permanent strains. The reloading phase shows the presence of hysteresis loops. The above interpretations are reinforced by examining the variation of average sliding on microcracks faces. Two examples corresponding to microcracks orientations $\phi = 45°$; and $\phi = 62°$ are presented in figure 4.

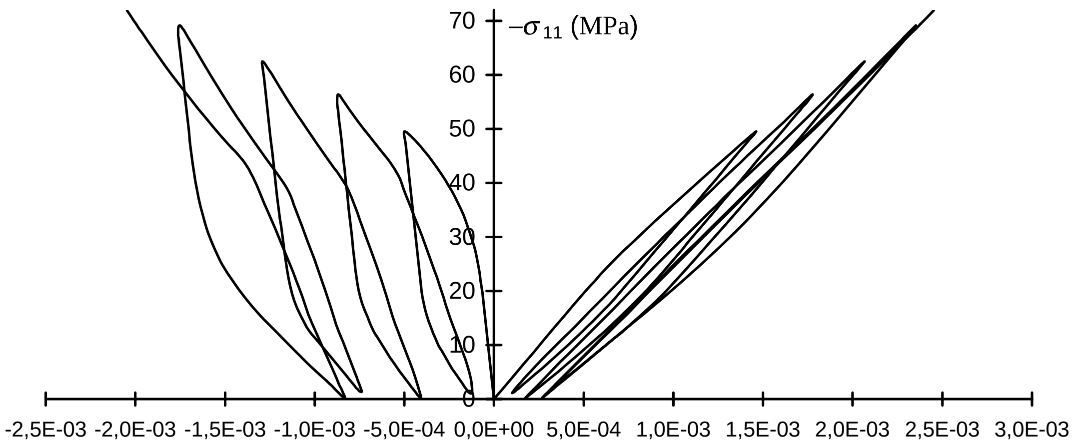


Figure 3 : *Simulation of the material response under uniaxial cyclic compression*

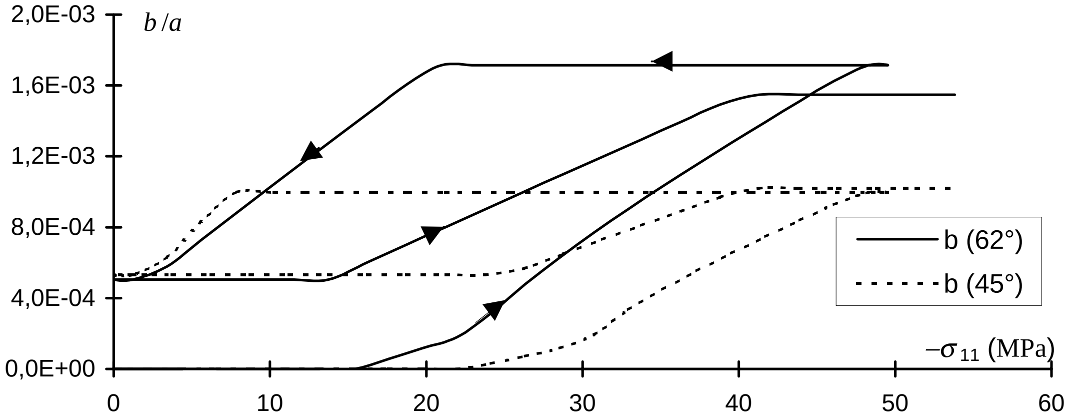


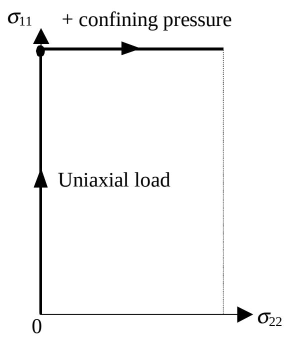
Figure 4: *Uniaxial cyclic compression : sliding evolution.*

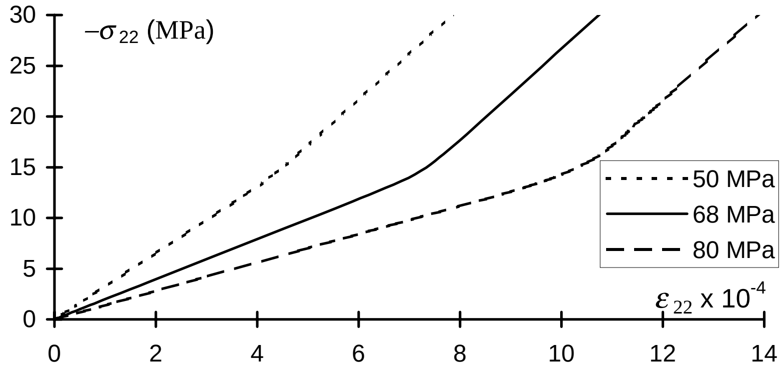
4.2 Damage deactivation during microcracks closure by confining :

A particular phenomenon observed in brittle materials is the deactivation of damage for some stress paths (Chaboche, 1993). This effect, which occurs when progressive microcracks is observed, is studied here by increasing the confining pressure after a damaging uniaxial compression (see figure 5a). Figure 5b shows the variation of lateral strain during the confining step, for three previous damaging stress levels. For low confining pressure, the material behaves almost linearly with different compliances related to the previous damage. When all microcracks are closed, damage is completely deactivated and the materials responds as in the initial state.

5 CONCLUSION

A three dimensional micromechanical model, based on a frictional sliding crack and on a branching mechanism, is presented in a rate form. Application of this model to the study of a brittle rocks show it’s capabilities to predict several important aspects of brittle anisotropic damage (oriented mesocracks growth). Particularly, hysteretic loops and apparition of permanent stress due to friction phenomena are well described. Damage deactivation, due to progressive closure of microcracks by confining, is also predicted by the model.





a) b)

Figure 5 : Microcracks closure by increasing of confining pressure - *Number on figure b) indicates the previous uniaxial compression* a) stress path b) Material response

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